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Gravitational wave-frequencies and energies in hypernovae

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ABSTRACT

A torus develops a state of suspended accretion against a magnetic wall around a rapidly rotating black hole formed in core-collapse hypernovae. It hereby emits about 10% of the black hole spin-energy in gravitational radiation from a finite number of multipole mass moments. We quantify the relation between the frequency of quadrupole gravitational radiation and the energy output E_w in torus winds by $f_{gw} \simeq 470\text{Hz}(E_w/4 \times 10^{52}\text{erg})^{1/2}(7M_\odot/M)^{3/2}$, where M denotes the mass of the black hole. We propose that E_w irradiates the remnant stellar envelope from within. We identify E_w with energies $\sim 10^{52}$ erg inferred from X-ray observations on matter injecta; and the poloidal curvature in the magnetic wall with the horizon opening angle in baryon poor outflows that power true GRB energies of $E_\gamma \simeq 3 \times 10^{51}$ erg.

Subject headings: black hole physics — gamma-rays: bursts and theory – gravitational waves

1. Introduction

Stellar mass black holes surrounded by a torus of a few tenths of a solar mass may form in prompt core-collapse of young massive stars by conservation of mass and angular momentum, as in the Woosley-Paczynski-Brown scenario of hypernovae (Woosley 1993; Paczyński 1998; Brown et al. 2000). These are transient systems which are interesting from the point of view of the first law of thermodynamics. A torus surrounded a rapidly rotating black hole develops a suspended accretion state for the lifetime of rapid spin of the black hole (van Putten 2001) – generally a secular timescale. The torus catalyzes some 10% of black hole spin-energy into gravitational radiation, with additional winds, thermal and MeV neutrino emissions (van Putten & Levinson 2002). Gravitational radiation is emitted by a finite number of mass multiple moments produced by a Papaloizou-Pringle instability (Papaloizou & Pringle 1984;

Goldreich et al. 1986; van Putten 2002); torus winds form analogously to pulsar winds, and MeV temperatures develop by internal heating due to shear in response to competing torques acting on the inner and the outer face of the torus. Similar considerations apply to black hole-torus systems formed in black hole-neutron star coalescence, provided that the black hole spins rapidly.

The predicted gravitational radiation may be determined by upcoming gravitational wave experiments LIGO (Abramovici et al. 1992), VIRGO (Bradaschia et al. 1992), TAMA (Masaki et al. 2002) and others, possibly in combination with any of the bar or sphere detectors presently being developed. Accurate prediction of the gravitational wave-frequency is important in estimating the expected signal to noise ratios (e.g., Cutler & Thorne (2002)), or in the design of dedicated experiments, such as dual recycling interferometry (Harry et al. 2002).

In this *Letter*, we quantify the relation between the gravitational wave-frequency and the associated energetics in torus winds. In a hypernova, these wind energies deposit their momenta onto surrounding remnant envelope, providing enhanced kinetic energies to matter ejecta. Wind energies determined from observational data on kinetic energies in GRB-supernova events (e.g., (Reeves et al. 2002)) and their remnants hereby provide a means for constraining the gravitational wave-frequency, and circumventing uncertainties in model parameters.

The energetics of black hole-torus systems in suspended accretion depends on the mass of the black hole and the angular velocity of the torus. Hypernovae are believed to involve stellar mass black holes of about $4 - 14M_{\odot}$, consistent with the observed mass distribution in soft X-ray transients. The angular velocity defines the efficiency of the catalytic conversion of black hole spin-energy as well as the frequency of gravitational radiation (at various multipole moments). Yet, it is difficult to determine from first principles, and it becomes of interest to consider ways to constrain the angular frequency of the torus from the output in torus winds. The specific approach presented here considers the phenomenology of torus winds, as well as a detailed calculation on the energy output in gravitational radiation, winds and thermal and neutrino emissions.

The fate of the torus wind energies is determined by its nearby environment. We here propose that the torus wind energy is largely deposited within and onto the surrounding remnant stellar envelope, in the form of radiation and kinetic energy, respectively. We thus attribute the X-ray line-emission energies in matter ejecta in GRB-supernova associated events (e.g., (Reeves et al. 2002; Ghisellini et al. 2002)) to irradiation *from within*, as the expanding envelope reaches optical depth of order unity. We note that the deposition of torus wind energies is essentially that of a point explosion, given the relatively short durations of

activity at hand (about 20 s in association with a long gamma-ray burst).

It has further been suggested that hypernovae may be associated with supershells (Efremov et al. 1998; Kim et al. 1999) which are X-ray bright (Wang 1999). Indeed, these X-ray bright features appear to be distinct from collective supernovae (Bunne et al. 2001). A baryonic component to torus winds represents ejecta, which may further account for chemical enhancements in the companion stars in some of the soft X-ray transients (Brown et al. 2000), e.g., GRO J1655-40 (Israelian et al. 1999) and V4148Sgr (Orosz et al. 2002).

A GRB-hypernova association is receiving increasing support from the association of long gamma-ray bursts with supernovae, as in notably in GRB 980425/1998bw (Galama et al. 1998) and, more recently, GRB 011121 (Bloom et al. 2002), GRB 011211 (Reeves et al. 2002) and GRB 020405 (Price et al. 2002). Indeed, long GRBs are associated with star-forming regions (Bloom et al. 2002), and hence possibly with young massive stars in binaries. This appears to be consistent with a host environment in molecular clouds (Price et al. 2002). In turn, this suggests closer consideration of an association of hypernovae and their remnants in molecular clouds.

Calorimetry on systems consisting of a torus in suspended accretion around a rapidly spinning black hole therefore can be approached as follows. First, the beamed output in true GRB energies E_γ has been determined to be a few times 10^{50} ergs (Frail et al. 2001), which is indicative of ultrarelativistic baryon poor jets of somewhat larger energies. Second, the energy E_w in torus winds can be determined from energies associated with matter ejecta and their remnants in the host environment, the impact on a binary companion star in a remnant soft X-ray transient, and from beaming in GRBs when derived from collimation by winds. Third, the energy emitted in gravitational radiation of $E_{gw} \simeq 6 \times 10^{53}$ erg for a ten solar mass extreme Kerr black hole may be determined by upcoming gravitational wave experiments. A fourth energy deposition in MeV neutrinos is unlikely to be detectable in the foreseeable future, given the relatively large distances of these transient events. The energies E_w and E_{gw} are both produced by the torus and are a function of its angular velocity. This suggests determining E_w in an effort to predict E_{gw} and hence the frequency f_{gw} of gravitational radiation, as E_{gw} and f_{gw} are closely related.

We shall derive simplified expressions for the fractions E_{gw}/E_{rot} , E_w/E_{rot} and in dissipation E_{diss}/E_{rot} , relative to the rotational energy E_{rot} of the black hole. These results are used to define an observational constraint on the torus-to-black hole angular velocity by the energy emitted in torus winds. This serves to circumvent otherwise uncertain physical parameters, notably the viscosity in the torus which provides coupling between the inner and the outer face. We propose to determine E_w from calorimetry on hypernova ejecta and their remnants. Accurate prediction of the gravitational wave spectrum from wind energies will

be useful in designing dedicated experiments for searches for gravitational radiation from hypernovae.

2. Multipole moments in wide tori

Quite generally, a torus tends to develop instabilities in response to shear. This can be studied analytically in the approximation of incompressible fluid about an unperturbed angular velocity

$$\Omega = \Omega_T (a/r)^q, \quad (1)$$

where $q \geq 3/2$ denotes the rotation index and $\Omega_T \simeq M^{1/2}/a^{3/2}$ (Papaloizou & Pringle 1984). In the inviscid limit, we are further at liberty to consider the evolution of irrotational modes in response to initially irrotational perturbations to the underlying flow (vortical if $q \neq 2$) by Kelvin’s theorem. This approach shows the Papaloizou-Pringle instability (Papaloizou & Pringle 1984) to also operate in wide tori (van Putten 2002). The neutral stability curves of the resulting buckling modes can be described in terms of the critical rotation index $q_c = q_c(b/a, m)$ as a function of the minor-to-major radius b/a . Quadratic fits to these stability curves are

$$q_c(b/a, m) = \begin{cases} 2.49(b/a)^2 + 1.73 & (m = 2) \\ 6.47(b/a)^2 + 1.73 & (m = 3) \\ 12.4(b/a)^2 + 1.73 & (m = 4) \\ 20.0(b/a)^2 + 1.73 & (m = 5) \\ 0.85m^2(b/a)^2 + 1.73 & (m > 5) \end{cases} \quad (2)$$

Instability sets in above these curves, stability below. We note that for $m = 2$ the critical value $q_c = 2$ obtains for

$$b/a = 0.3225, \quad (3)$$

associated with the Rayleigh stability criterion for the azimuthally symmetric wave mode $m = 0$. For large m , we use the numerical result of critical values $b/a = 0.56/m$ for $q = 2$.

A quadrupole buckling mode radiates gravitational waves at close to twice the angular frequency of torus (van Putten 2002). Because the buckling mode represents an internal flow of energy and angular momentum from the inner to the outer face of the torus, in which total energy and angular momentum of the wave remain zero, it is not subject directly to the Chandrasekhar-Friedman-Schutz instability. Nevertheless, it is stimulated by gravitational radiation-backreaction forces, as derived in the approximation of the Burke-Thorne potential (van Putten 2002).

3. Energy output from a torus in suspended accretion

The suspended accretion state is described by balance of energy and angular momentum, pertaining to gravitational radiation, Poynting flux-dominated winds, thermal and neutrino emissions (van Putten 2001):

$$\begin{cases} \tau_+ = \tau_- + \tau_{rad}, \\ \Omega_+ \tau_+ = \Omega_- \tau_- + \Omega_T \tau_{rad} + P_d, \end{cases} \quad (4)$$

where P_d denotes dissipation in thermal and neutrino emissions, $\Omega_T = (\Omega_+ + \Omega_-)/2$ is the angular velocity of the torus, defined as the mean of the angular velocities of the inner and outer faces with angular velocities Ω_{\pm} . The inner and outer faces are subject to Maxwell stresses, giving rise to torques τ_+ (ingoing) and τ_- (outgoing). In what follows $2\pi A$ shall denote the active magnetic flux supported by the torus, representing open magnetic field-lines connected to infinity and the horizon of the black hole. These are endowed with no-slip/slip boundary conditions, and thereby assume the angular velocity of the torus. The remainder of magnetic field-lines are inactive, making up a toroidal ‘bag’ of closed field-lines, both on the inner and the outer face, similar to the closed field-lines in pulsar magnetospheres (Goldreich & Julian 1969; van Putten 2001). Thus, $\tau_- = A^2 f_w^2 \Omega_-$ associated with a fraction f_w of flux in torus winds to infinity, and $\tau_+ = (\Omega_H - \Omega_+) A^2 f_H^2$ associated with a fraction f_H of torus winds entering the black hole. With this definition, $f_w + f_H = 1$.

We consider shear stresses and the resulting dissipation in the torus to be due to turbulent magnetohydrodynamical flow, in response to competing torques acting on the inner and outer face. These competing torques promote super-Keplerian and sub-Keplerian motions, and hence the torus assumes a geometrically thick shape. By dimensional analysis, we consider the constitutive relation $P_d = \gamma A_r^2 (\Omega_+ - \Omega_-)^2$ with γ a factor of order unity and $A_r = Rh < B_r^2 >^{1/2}$ denoting the root mean square of the radial flux averaged over the interface between the two faces with contact area $2\pi Rh$. The detailed structure of the magnetohydrodynamical flow which develops determines the ratio $\gamma A_r^2 / A^2$, which parametrizes the effective viscosity. The constitutive law for P_d thereby contains one free parameter. In what follows, we parametrize the relative strength of the net radial magnetic field as

$$z = \left(\frac{b}{a} \right) \left(\frac{\gamma A_r^2}{A^2} \right), \quad (5)$$

where a and b denote the major and the minor radius of the torus, respectively. This choice of parametrization (5) is such that z becomes independent of the aspect ratio b/a , whenever the magnetohydrodynamical flow develops a flat infrared spectrum up to the first geometrical break $m^* = [a/b]$ in the azimuthal wave-number.

A leading order expansion in the minor radius b of (1) obtains $\Omega_{\pm} = \Omega_T(1 \pm \delta)$ with $\Omega_a = \Omega_T$ denoting the mean angular velocity of the torus and Ω_{\pm} the average angular velocity of the inner and the outer face. Here,

$$\delta = \frac{qb}{2a} \quad (6)$$

denotes the slenderness ratio of the torus. In particular, we have $[\Omega] \simeq \Omega_+ - \Omega_- \simeq q\Omega_T b/a$. The equations (4) can be now be solved for

$$\eta = \frac{\Omega_T}{\Omega_H}, \quad (7)$$

and the energy output E_{gw} in gravitational radiation. Explicit expressions for the energy output E_w and E_d in winds and the combined dissipation in heat and neutrino emissions. The model parameters are, therefore, the rotation index q , the dimensionless magnetohydrodynamical viscosity z , the slenderness ratio δ , the ratio of angular velocities η , in addition to flux fractions f_H and f_w .

3.1. Estimate of η

The first equation of (4) may be used to eliminate τ_+ in the second equation, $\Omega\tau_{rad} = \Omega_+\tau_+ - \Omega_-\tau_- - P_d$, which obtains

$$P_d = \frac{1}{2}[\Omega]\tau_{rad} + [\Omega]\tau_-. \quad (8)$$

With the constitutive ansatz for P_d given above, it follows that $\tau_{rad} = 2A_r^2[\Omega] - 2\tau_-$. The luminosity in gravitational becomes $L_{gw} \simeq \Omega_T\tau_{rad}$, in view of the fact that the lower order multipole moments are essentially in corotation with the torus. Here,

$$\Omega_T\tau_{rad} = \Omega^2 A^2 \left[2 \left(\frac{A_r^2}{A^2} \right) \left(\frac{[\Omega]}{\Omega_T} \right) - 2f_w^2 \right] = \alpha \Omega_T^2 A^2, \quad (9)$$

where $\alpha = 2qz - 2f_w^2$ by $[\Omega]/\Omega_T = qb/a$. With forementioned expression for τ_+ , the first equation of (4) with (9) obtains $(\Omega_H - \Omega_+)f_H^2 = f_w^2\Omega_- + \alpha\Omega$. Writing $\Omega_{\pm} = \Omega_T \pm [\Omega]/2$, we obtain

$$\eta = \frac{f_H^2}{\alpha + f_H^2 + f_w^2 + \delta(f_H^2 - f_w^2)}. \quad (10)$$

3.2. Estimate of E_{gw}

An expression for the luminosity L_{gw} in gravitational radiation may be obtained, upon noting that most of the black hole luminosity is irradiated into the torus, i.e.: $L_H \simeq \Omega_+ \tau_+$. (Output along an open flux-tube by the black hole in association with a GRB is subdominant; van Putten & Levinson (2002).) We write $\Omega_T^2 f_H^2 = \eta \Omega \Omega_H f_H^2 A^2 = \eta (\Omega_T/\Omega_+) \Omega_+ \tau_+ \Omega_H / (\Omega_H - \Omega_+)$ for substitution in (9) after multiplication by α/f_H^2 . By (10) with $\Omega_+ = \Omega_T(1 + \delta)$, we have $L_{gw}/L_H = \alpha^{-1} f_H^2 \eta / (1 + \delta - \eta(1 + \delta)^2)$, i.e.: $L_{gw}/L_H \simeq \alpha / (\alpha + f_w^2 + \delta(\alpha + f_H^2 + f_w^2))$, upon neglecting terms of the order $\eta\delta$ and higher. The total rate of conversion of rotational energy of the black hole, including dissipation in the horizon (Thorne et al. 1986), is given by $\dot{E}_H = \Omega_H(\Omega_H - \Omega_+) f_H^2 A^2$ over a duration $T = E_{rot}/\dot{E}_H$. The ratio of energy released in gravitational radiation as a fraction of the rotational energy E_{rot} , therefore, becomes

$$\frac{E_{gw}}{E_{rot}} = \frac{\alpha\eta}{\alpha + f_w^2 + \delta(\alpha + f_H^2 + f_w^2)}. \quad (11)$$

4. Energy estimates for symmetric flux distribution

Consider a symmetric flux-distribution, given by equal fractions of open magnetic flux on the inner and the outer face:

$$f_H = f_w = 1/2. \quad (12)$$

Writing (10) in the form of $\eta = f_H^2 / (2qz + (1 + \delta)(f_H^2 - f_w^2))$, it reduces to $\eta = 1/8qz$. A fiducial value of $\eta = 0.15$ obtains by considering the Rayleigh value $q = 2$ and $z = 0.56$, which attributes the radial modes in the infrared magnetohydrodynamical spectrum to an equipartition between the unstable Papaloizou-Pringle buckling modes. The estimate (11) becomes

$$\frac{E_{gw}}{E_{rot}} = \frac{8qz - 2}{8qz(8qz(1 + \delta/4) - 1)}. \quad (13)$$

The energy released in winds satisfies $E_w = \alpha^{-1} f_w^2 (1 - \delta)^2 E_{gw}$, whereby

$$\frac{E_w}{E_{rot}} = \frac{(1 - \delta)^2}{8qz(8qz(1 + \delta/4) - 1)}. \quad (14)$$

Finally, (8) shows that the energy radiated by dissipation in the torus satisfies

$$\frac{E_d}{E_{rot}} = \delta \frac{E_{gw}}{E_{rot}} + \frac{2\delta}{1 - \delta} \frac{E_w}{E_{rot}}. \quad (15)$$

The case of strong viscosity (large z) and small δ reduce these expressions further to the three fractions

$$E_{gw}/E_{rot} \sim \eta, \quad E_w/E_{rot} \sim \eta^2, \quad E_d/E_{rot} \sim \eta\delta. \quad (16)$$

In association to GRBs, we attribute a minor output of black hole-spin energy with the formation of a baryon poor jet of energy E_j along an open magnetic flux-tube of the black hole. A universal horizon half-opening angle defines $E_j \sim \theta_H^4$ to be standard, and a generally small fraction of rotational energy of the black hole (van Putten & Levinson 2002). The simplest geometrical relationship obtains when θ_H is given by the poloidal curvature of the magnetic field, i.e., $\theta_H \simeq 10^\circ(6M/a)$. Without fine-tuning, this gives a small fraction $E_j/E_{rot} \sim 10^{-3}$ released in baryon poor outflows, accompanied by a spread of one order of magnitude in response to a spread in torus radius by a factor of two – in agreement with the observed GRB energies E_γ (Frail et al. 2001). The associated wind energies $E_w/E_{rot} \propto (M/a)^3$ vary somewhat less.

5. Frequency estimate of quadrupole radiation

The expressions (16) provide a link between the energy E_w in torus winds and the frequency in quadrupolar gravitational radiation, set by the angular velocity η of the torus. We propose to estimate η from E_w , to circumvents uncertainties in qz . This gives a frequency of quadrupole gravitational radiation given by $f_{gw} \simeq 1/(2\pi M)(E_w/E_{rot})$, or

$$f_{gw} \simeq 470\text{Hz} \left(\frac{E_w}{4 \times 10^{52}\text{erg}} \right)^{1/2} \left(\frac{7M_\odot}{M} \right)^{3/2}. \quad (17)$$

The wind-energy scale of $4 \times 10^{52}\text{erg}$ corresponds to $\eta = 0.1$ and $M = 7M_\odot$. We can refine (17) by including a factor $(1 - 2\eta)/(1 - \delta/2)$, in view of (13) and (14). Higher order multipole moments are present for a torus of increasingly smaller width (see §2), which emit at commensurably higher frequencies, and probably at somewhat lower intensities.

6. Calorimetry on ejecta and hypernova remnants

Torus wind energies impact the remnant stellar stellar envelope from within. This results in enhanced kinetic energies in matter ejecta in GRB-supernova associated events and, ultimately, impact the host environment. For GRB 011211 (Reeves et al. 2002), we

have

$$E_w \simeq m_{ej}\beta_{ej} \simeq 2 \times 10^{52} \left(\frac{\theta_{ej}}{20^\circ} \right)^2 \text{ erg} \quad (18)$$

for ejecta of mass m_{ej} with half-opening angle θ_{ej} and $\beta_{ej} = v_{ej}/c$, where $v_{ej} = 0.1c$ is the velocity of the ejecta in terms of the velocity of light c . At present, the angle θ_{ej} is not constrained observationally. It may be larger than the GRB beaming angle, in which case we should see an over abundance of hypernovae with dim or no GRBs. We remark that only beamed outflows – baryon poor from the black hole – have sufficiently high luminosity density per steradian to plow through a stellar envelope (MacFadyen & Woosley 1998).

As the envelope expands, its optical depth reaches order unity and is irradiated from within by the accumulated remnant radiation from the torus wind energies. We attribute the observed X-ray line-emissions to this continuum emission, whose energies are on the order of 10^{52} erg (Ghisellini et al. 2002).

Follow-up calorimetry may be pursued on hypernova remnants, possibly on X-ray bright supershells. This could be pursued by comparing this population in a nearby galaxy with with the true GRB-hypernova event rate of about 5×10^{-4} times the supernova rate (Woods & Loeb (1998), including beaming), and identification of X-ray point sources (see Brown et al. (2000)).

7. Conclusions

We quantify the energetics of a torus in suspended accretion against a magnetic wall around a rapidly rotating black hole formed in hypernovae. The torus catalyzes most of the black hole-spin energy, radiating a major fraction in a band limited burst of gravitational radiation. The frequency sweep is about ten percent during the conversion of half the rotational energy of an extreme Kerr black hole. The durations should agree with the redshifted distribution of T_{90} of long GRBs (Fig. 1 in van Putten (2002)).

In the slender torus limit, simplified expressions are derived for energies emitted in gravitational radiation, winds and dissipation in the torus. We attribute the narrow distribution in observed true gamma-ray energies to baryon poor jets produced in an open magnetic flux-tube on the black hole to geometry without fine-tuning: the fraction of rotational energy released in baryon poor outflows is set by a horizon half-opening angle θ_H given by the poloidal curvature of the magnetic wall.

The frequency of gravitational radiation is strongly correlated to the energy output in torus winds by (17). We attribute the energies associated in matter ejecta in GRB-supernova

events to the deposition of torus wind energy within and onto the surrounding remnant stellar envelope. We propose to hereby constrain the expected frequency of gravitational radiation by calorimetry on these matter ejecta and their remnants in the host environment. The latter may be pursued by statistics on the kinetic energies in matter ejecta, and by calorimetry on hypernova remnants in association with molecular clouds, conceivably in the form of X-ray bright supershells.

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REFERENCES

- Abramovici, A., et al., *Science*, 256, 325
- Bloom, J.S., Kulkarni, S.R., & Djorkovski, S.G., 2002, *AJ*, 123, 111
- Bloom, J.S., et al., 2002, *ApJ, Lett.*, submitted
- Bradaschia, C., et al., *Phys. Lett. A.*, 163, 15
- Brown G.E., Lee, C.-H., Wijers, R.A.M.J., Lee, H.K., Israelian, G., & Bethe, H.A., 2000, *NewA*, 5, 191
- Bunne, B.C., Points, S.D., & Chu, Y.-H., 2001, *ApJ*, 136, 119
- Cutler, C., & Thorne, K.S., 2002, *gr-qc/0204090*
- Lee, C.-H., Brown, G.E., & Wijers, R.A.M.J., 2002, submitted
- Efremov, Y.N., Elmegreen, B.G., & Hodge, P.W., 1998, *ApJ*, 501, L163
- Frail, D.A., et al., 2001, *ApJ*, 567, L47
- Galama, T.J., et al., 1998, *Nature*, 395, 670
- Ghisellini, G., Lazatti, D., Rossi, E., & Rees, M.J., 2002, *A&A*, L33
- Goldreich, P.M., & Julian, W.H., 1969, *ApJ*, 157, 869
- Goldreich, P., Goodman, J., & Narayan, R., 1986, *MNRAS*, 221, 339
- Harry G.M., Houser J.L., & Strain K.A., 2002, *Phys. Rev. D.*, 65, 082001

- Israelian, G., et al., 1999, *Nature*, 401, 192
- Kim, S., Dopita, M.A., Staveley-Smith, L., & Bessel, M.S., 1999, *ApJ*, 118(6), 2797
- MacFadyen, A., & Woosley, S.E., 1998, *ApJ*, 524, 262
- Orosz, J.A., et al., 2001, *ApJ*, 555, 4890
- Paczynski, B.P., 1998, *ApJ*, 494, L45
- Papaloizou, J.C.B., & Pringle, J.E., 1984, *MNRAS*, 1984, 208, 721
- Peter and Mathews, 1963, *Phys. Rev.*, 131, 435
- Price, P.A., et al., 2002, astro-ph/0208008, submitted to *ApJ Lett.*
- Reeves, J.N., et al., 2002, *Nature*, 416, 512
- Masaki, A., et al., 2001, *Phys. Rev. Lett.*, 86(18), 3950
- Thorne, K.S., 1969, *ApJ*, 158, 1; 1969, *ibid.* 158, 997
- Thorne, K.S., Price, R.H., & Price, D.A., 1986, *Black holes: The membrane paradigm* (Yale University Press, New Haven, CT)
- van Putten, M.H.P.M., 2001, *Phys. Rep.* 345, 1
- van Putten M.H.P.M. & Levinson, A., 2002, *Science*, 295, 1874; published online 21 February 2002; 10.1126/science.1068634
- van Putten, M.H.P.M., 2002, *ApJ*, 575, L71
- Wang, Q.D., *ApJ*, 517, L27
- Woods, E., & Loeb, A., 1998, *ApJ*, 508, 760
- Woosley, S.E., 1993, *ApJ*, 405, 273